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# Low flow frequency analysis by L-moments method (Case study: Iranian Central Plateau River Basin)

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#### Abstract

Knowledge about low flow statistics is essential for effective water resource planning and management in ungauged or poorly gauged catchment areas, especially in arid and semi-arid regions such as Iran. We employed a data set of 20 river flow time-series from the Iranian Central Plateau River Basin, Iran to evaluate the low-flow series using several frequency analysis methods and compared the result of these methods in their ability to set low flows for the conservation of instream water uses. Theoretical frequency distributions including the log-normal, three-parameter lognormal, Gumbel, Pearson type III, and log Pearson type III were applied with the low-flow series. Goodness-of-fit tests including Lmoment and conventional moment methods for the observed data were applied to identify the best distributions. For each distribution, the calculated values of the residual sum of squares (RSS) was applied to compare between the conventional moment and L-moment methods, and the best method was selected to determine the most appropriate probability distribution. The lowest RSS values were used to select the best distribution for each station. Then, T-year low-flow series was estimated using the best probability distribution. Our results suggested that, for annual low flows, based on the computed RSS, Pearson type III, log Pearson type III, Gumbel distributions, and L-moment method were suitably distinguished for 85%, 10%, and 5% of the stations, respectively. Finally, Compared to the conventional moment method, L-moments method was found to be more adequate to identify low-flow series probability distributions in the Iranian Central Plateau River Basin, while Pearson type III was found to be the best probability distribution for modeling minimum flow series in the study area.

Keywords: Arid and semi-arid region; Frequency analysis; L-moments; Low flow

# 1. Introduction

Accurate estimates and frequency analysis of low flow are required for a range of objectives in water resource management and engineering, including environmental flow requirements, water uses, water discharges into streams, and hydropower operation (Smakhtin, 2001; Gustard *et al.*, 2004; Laaha and Bloschi, 2006). Low flow frequency analysis is best evaluated from an

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observed stream flow data by determining the magnitude of hydrological variables corresponding to a given set of frequencies or recurrence intervals (McCuen, 2003).

Several goodness-of-fit methods exist to select suitable probability distributions for stream flow sequences. Since its introduction (Hosking, 1990), numerous studies have suggested L-moments for the estimation of the goodness-of-fit of different probability distributions to a regional data (Chowdhury *et al.*, 1991; Hosking and Wallis, 1993; Stedinger *et al.*, 1993; Vogel and Fennessy, 1993; Vogel and Wilson, 1996; Parida *et al.*, 1998; Pandey *et al.*, 2001; Kumar *et al.*, 2003; Lim and Lye, 2003; Salajegheh *et al.*, 2008; Shi *et* 

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al., 2010; Keshtkar et al., 2012; Rostami, 2013). L-moments method was used much less frequently for low flow frequency analysis than on-flood flow frequency analysis (Eslamian and Chavoshi, 2003; Malekinezhad et al., 2011). Pearson (1995) applied the L-moment diagram for nearly 500 catchment areas to search for suitable probability distributions in order to model the annual minimum flow series in New Zealand.

Vogel and Wilson (1996) evaluated the probability distributions of low-flow series at the national scale in the USA. Delleur et al. (1988) and Vogel and Kroll (1989) reviewed several studies comparing the fit of alternative probability distributions and parameter estimation method to a sequence of annual minimum flow series. Caruso (2000) applied L-moments for evaluating the goodness-of-fit as well as to select the best distributions of minimum flows in 21 rivers of the Otago region of the South Island of New Zealand and found that the GEV distribution was the best probability distribution for the group of sites in Otago. Kroll and Vogel (2002) applied the Lmoment diagram to recognize the probability of low-flow series in the USA and suggested that the Pearson III (PIII) and LN3 were the suitable probability distributions for low flows in the USA. Minocha (2003) suggested that a suitable probability distribution should be selected using both the L-moment diagram and the goodness-offit measure, as described by Hosking and Wallis (1997).

Bayazit and Onoz (2002) used LL-moment to predict the parameters of the different probability distributions fitted to the 36-years record of 7-day annual minimum flows in the Trent River basin, UK and then compared the results obtained with those from L-moment method. Yurekli et al. (2005) used L-moment ratio diagram to estimate the parameters of the selected probability distributions of low flows in the Cekerek watershed of Turkey. Chen et al. (2006) showed that the results of regional frequency analysis of low flows using L-moment ratios diagram may satisfy the practical application and that the threeparameter lognormal distribution was the best probability distribution for Dongjiang River in Hong Kong.

Peng et al. (2009) demonstrated that the low flow analysis by L-moments in the karst area of the southwest China was in a good agreement and that the generalized logistic (GLO) probability distribution was the most appropriate distribution. Shi et al. (2010) also applied L-moments for

regional frequency analysis to analyze low flow in the karst area of southwest China. Their results indicated that, based on the L-moments ratio diagram  $Z^{DIST}$  and  $t_4$  statistic criteria, GLO distribution was a robust distribution for the study area. Dodongeh et al. (2013) used L-moment for low-flow frequency analysis at ungauged sites in Sefidroud basin of northwestern Iran. Uncertainty analysis of model parameters and low-flow estimations was conducted using the Bayesian inference. The results showed that the L-moment method provided reasonably good accuracy for atsite as well as ungauged site frequency analysis and that GLO and Pearson type III distributions were the best-fit regional model for the east and west regions, respectively.

Recently, other approaches were also applied to investigate and analyze low-flow series. Among which, Sawaske and Freyberg (2014) focused on the identification and computation of indices representative of annual low-flow conditions and base-flow recession form. Their results showed that, in the past 40-80 years, widespread trends of increasing rates of base-flow recession and decreasing annual low-flow conditions existed throughout the region. Staudinger and Seibert (2014) assessed the relative importance of the current hydrological state and weather during the prediction period and found that the maximum detectable influences of initial conditions ranged from 50 days to at least 1 year and that drier initial conditions of soil moisture and groundwater as well as more initial snow resulted in longer effects of the initial conditions. Du et al. (2015) improved the characterization of nonstationary return period and risk under the ENE interpretation by employing meteorological covariates in the nonstationary frequency analysis.

The downscaled meteorological variables from the General Circulation Models (GCMs) and traditional approach were used for the analysis of annual minimum monthly stream flow series of two stations in the Wei River, China, which evaluated the nonstationary return period and the various significant risks from the stationary case. They concluded that the return period and risk analysis of nonstationary low-flow series may be suitable for water resource management during the dry seasons, exacerbated by climate change. Thomas *et al.* (2015) determined the low-flow predictors for 16 small catchment areas in the Northeast Germany along with their long-term shifts between 1965 and 2006. Non-linear

regression models (support vector machine regression) were calibrated to iteratively select the most suitable low-flow predictors regarding the annual 30-day low flow. Their results showed that changes in the relevant processes or flow paths generated low flows. In addition, their results proved that the determined predictors, temporal patterns, and patterns between the catchments support the development of low-flow monitoring systems and facilitate identification of catchment areas where the adaption measures aimed more at increasing the groundwater recharge.

Low flows became a critical issue in water resource researches studied in the past two decades, including low-flow frequency analysis, base-flow separation, characterization of stream flow recessions, and low-flow estimation in ungauged river catchments in the arid and semiarid regions of Iran (Eslamian and Feizi, 2007; Sarhadi et al., 2008; Modars, 2008; Modares, 2009; Dodongeh et al., 2013; Rostami, 2013). Eslamian and Feizi (2007) applied L-moments for the selection of parent distributions to fit the maximum monthly rainfall data of some regions in the Zayandehrood Basin in Iran. Modarres (2008) performed a regional low-flow frequency analysis in the north of Iran using L-moments. Later, Modarres (2009) studied the spatial variation and regional frequency distribution of annual maximum dry spell length for the Isfahan Province in Iran using both L-moment and multivariate analysis. The purpose of the present study was to select the suitable probability distributions for sequences of annual minimum stream flow using L-moment diagrams for 20 gauged catchments in the Iranian Central Plateau River Basin.

# 2. Materials and Methods

#### 2.1. Study area and data

The average annual precipitation of Iran is approximately 240 mm, which is about 30% of the worlds' average. The growing water demand has also caused a reduction in the annual per capita water resources. The uneven distribution of water throughout the country and the population growth rate have caused the current water shortages in the main regions of the country, particularly in the central, eastern, and the southeastern regions. The Ministry of Energy has divided the country into six major hydrological basins. The present study was conducted in the Iranian Central Plateau River Basin, which included arid and semi-arid climates, where the water availability demonstrated strong seasonal and inter annual fluctuations. Every year, approximately 80% of the annual rainfall and runoff occur during the 4 months between February and May, while low flows are normally observed during the remaining 8 months. In this study, 20 gauging hydrometric stations in representative locations throughout the Iranian Central Plateau River Basin were applied for lowflow analysis (Fig. 1).

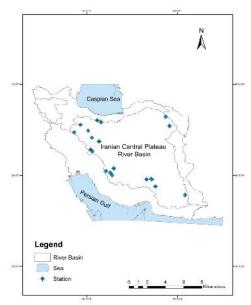


Fig. 1. Location of gauging stations used for low-flow analysis in the Iranian Central Plateau River Basin

These sites were selected as they were considered important from a management perspective and also because most of their low-flow series were considered reliable. There are no important artificial influences in these catchment areas, and the selection was therefore based on the Ministry of Jihade–Keshavarzi information, which indicated no significant abstraction in the

contributing catchments upstream of the selected stations. The record length of daily discharge values was about 25 years. The discharge data were collected by the Water Resources Management Company, Ministry of Energy of Iran. The selected hydrometric stations and their low-flow series are presented in Table 1 and Figure 2.

Table 1. Gauging stations used for low-flow analysis in the Iranian Central Plateau River Basin

Station	River	Longitude (E)	Latitude (N)	Altitude (m)	Catchment area (Km <sup>2</sup> )	Record (Year)
Ghamsar	Bonroud	51 22	33 44	1560	130	30
Abshanih	Yalfan	48 37	34 44	1980	165	41
Bandeh Abbasi	Veferghan	50 08	34 54	1120	13362	50
Abgarm	Kharroud	49 18	35 33	1560	2450	30
Sira	Karaj	51 09	36 03	1790	735	42
Roudak	Jajroud	51 33	35 51	1690	416	40
Dodahak	Ghomrood	50 34	34 08	1470	8851	50
GhaleShahrokh	Zaiandehroud	50 38	32 39	2100	1440	25
Eskandari	Polasjan	50 26	32 49	2130	1578	27
Chamriz	Kor	52 07	30 28	1800	3390	32
Dashtbal	Sivand	52 58	30 46	1660	6100	37
Polkhan	Kor	52 47	30 02	1590	6250	33
Safarzadeh	Halilrood	57 33	28 47	920	8420	25
Daman	Karvandar	60 48	27 50	720	3437	35
Jirofto	Haftkoosk	57 11	29 34	2600	237	35
Ghariatolarab	Chary	57 02	29 37	2190	173	37
Godarzarch	Abbakhsha	56 34	29 34	2200	1944	50
Bonkooh	Hablerood	52 35	30 18	1000	3209	45
Arieh	Bar	58 41	36 27	1390	119	45
Senobar	Shastdareh	59 06	35 26	1760	152	33

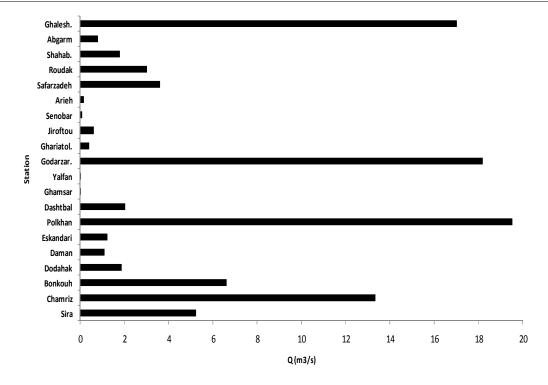


Fig. 2. Box plots of annual low-flow series for the selected stations

#### 2.2. Methods

The conventional moment and L-moment methods were used to estimate the probability distributions of low-flow series and return periods. The probability distributions including the log normal (LN), three-parameter lognormal (LN3), Gumbel (G), Pearson type III (P3), and log Pearson type III (LP3) were used with historical low-flow series.

## 2.2.1. L-moments Theory

The L-moment methods was a modification of the probability weighted moments (PWM) method presented by Greenwood et al. (1979). Compared to the conventional moments, the L-moments method offers theoretical advantages of being able to characterize a greater range of probability distributions and, when evaluated from a sample, of being more robust to outliers presence in the data. In addition, parameter evaluation with Lmoments is more accurate than even with the maximum likelihood evaluation for small samples (Hosking, 1990). The L-moment method and its ability in the goodness-of-fit tests has been wellexplained elsewhere (Vogel et al., 1993; Vogel and Wilson, 1996; Hosking and Wallis, 1997; Daviau et al., 2000; Peel et al., 2001, Smithers and Schulze, 2001; Jingyi and Hall, 2004; Eslamian and Feizi, 2007; Sarhadi et al., 2008; Shi et al., 2010; Keshtkar et al., 2012; Rostami, 2013).

# 2.2.2. Measures and tests of goodness-of-fit for comparing probability distributions

For the comparison of the probability distributions of fitting the data used in the present study, the RSS index, estimation of the relative goodness-of-fit were considered. The RSS was calculated by the formula given in Equation (1).

$$R.S.S = \left[ \frac{\sum_{j=1}^{n} (Q_e - Q_o)^2}{n - m} \right]^{1/2}$$
(1)

For each distribution, the calculated values of RSS was applied to compare the conventional moment with the L-moment as well as to select the best method to determine the most appropriate probability distribution. The lowest RSS values were used to select the best distribution for each station. Then, *T*-year low-flow series were estimated using the selected best probability distribution.

#### 3. Results and Discussions

To determine suitable probability distributions for frequency analysis of annual low flows in the 20 gauging stations studied throughout the Iranian Central Plateau River Basin, the first four L-moment's values and L-moment ratios were estimated, as given in Table 2.

Station	L-Moments				L-Moments Ratios			
Station	1	2	3	4	2	3	4	
Ghleshahrokh	10.9	3.3	0.5	-0.06	0.30	0.15	-0.02	
Abgarm	26.9	7.08	2.03	-0.17	0.26	0.29	-0.02	
Shahabasi	12	3.07	0.9	0.09	0.26	0.29	0.03	
Sira	34	9.8	2.1	-0.12	0.29	0.21	-0.01	
Roudak	2.2	0.3	0.2	0.1	0.14	0.67	0.33	
Chamriz	1.5	0.4	0.1	0.02	0.27	0.25	0.05	
Safarzadeh	1.5	0.3	0.1	0.05	0.20	0.33	0.17	
Arieh	1.6	0.3	0.15	0.06	0.19	0.50	0.20	
Senobar	34	8	3	0.4	0.24	0.38	0.05	
Ghariatolarab	0.04	0.01	0.001	0.0009	0.25	0.10	0.09	
Jirofto	2.8	0.5	0.3	0.06	0.18	0.60	0.12	
Daman	5	0.9	0.5	0.2	0.18	0.56	0.22	
Godarzarch	0.01	0.002	0.001	-0.0009	0.20	0.50	-0.45	
Eskandari	0.2	0.03	0.01	0.007	0.15	0.33	0.23	
Polkhan	0.1	0.02	0.01	0.003	0.20	0.50	0.15	
Bonkooh	3.1	0.8	0.2	0.04	0.26	0.25	0.05	
Dashtbal	0.6	0.1	0.05	0.02	0.17	0.50	0.20	
Dodehak	6.1	1.8	0.4	-0.04	0.30	0.22	-0.02	
Yalfan	8.3	0.8	0.7	0.7	0.10	0.88	0.88	
Ghamsar	0.53	0.1	0.04	0.01	0.19	0.40	0.10	

The values of L-moment ratios, r = r/2 for r = 3 lies between -1 and +1, and the L-coefficient of variation ( $L_{CV}$ ) satisfies 0 < 2 < 1 (Hosking, 1990). The greatest  $L_{CV}$  was 0.3 for the Dodehak and Ghaleshahrohk stations (Table 2). The highest value for both  $L_{sk}$  and  $L_{kur}$  was 0.88 and 0.88 for the Yalfan station. The high  $L_{cv}$  and  $L_{sk}$  values for Yalfan caused large discordancy that showed the

influence of outlier on the homogeneity and indicated that the best fitting distribution should be removed from the possible group of homogeneous stations within the Iran Central Plateau River Basin.

The graph of the theoretical distribution curves for  $L_{sk}$  and  $L_{kur}$  as well as the values for each station is shown in Figure 3.

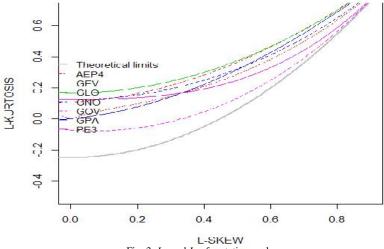


Fig. 3.  $L_{sk}$  and  $L_{kur}$  for stations values

The performance of the two parameter log normal (LN), three-parameter lognormal (LN3), Gumbel (G), Pearson type III (P3), and log Pearson type III (LP3) probability distributions used in this study for fitting the low-flow series from the 20 gauging stations were tested by RSS index.

Our results indicated that, the computed RSS, P3, LP3, and G distributions and L-moment method were suitably distinguished for 85%, 10%, and 5% of the stations, respectively for annual low flows (Table 3).

Table 3. Suitable probability distributions for low-flow series

Station	RSS	Suitable Probability Distribution
Ghleshahrokh	2.4	Р3
Abgarm	0.11	P3
Shahabasi	0.2	P3
Sira	0.57	LP3
Roudak	0.4	P3
Chamriz	2	P3
Safarzadeh	0.7	P3
Arieh	0.02	P3
Senobar	0.013	P3
Ghariatolarab	0.05	P3
Jirofto	0.08	P3
Daman	0.2	P3
Godarzarch	3.3	G
Eskandari	0.21	P3
Polkhan	2.8	P3
Bonkooh	0.9	LP3
Dashtbal	0.3	P3
Dodehak	0.32	P3
Yalfan	0.003	P3
Ghamsar	0.003	P3

Our results are in agreement with those of Matalas (1963), which is the only previous research that examined sequences of annual low flows. Most previous researches did not even recommend the P3 distribution for modeling annual low flows.

The estimated *T*-year low-flow series using the best of probability distribution are indicated in Table 4. For the two methods, the sum of scores

were estimated in order to select the best distribution. First rank was given to each distribution with lowest calculated RSS, fifth rank was given to each distribution with greatest calculated RSS, while equal scores were ranked similarly. Finally, the sum of scores estimated for any distribution with the lowest score was determined as the best distribution (Fig. 3).

Estimated		

Station	Return period (Year)						
Station	2	5	10	20	50	100	
Ghleshahrokh	9.8	11.6	12.6	13.2	14.1	14.6	
Abgarm	0.4	0.5	0.54	0.57	0.62	0.64	
Shahabasi	0.8	1	1.1	1.15	1.3	1.31	
Sira	3	3.7	4	4.4	4.8	5.1	
Roudak	1.8	2.1	2.25	2.36	2.52	2.61	
Chamriz	7.1	8.8	9.6	10.3	11.1	11.7	
Safarzadeh	0.88	1.3	1.55	1.7	1.94	2.1	
Arieh	0.03	0.04	0.05	0.05	0.06	0.065	
Senobar	0.03	0.04	0.04	0.04	0.04	0.05	
Ghariatolarab	0.11	0.15	0.17	0.18	0.2	0.21	
Jirofto	0.13	0.17	0.2	0.22	0.24	0.26	
Daman	0.3	0.4	0.5	0.5	0.56	0.59	
Godarzarch	0.46	1.3	2	2.5	11.6	17.7	
Eskandari	0.3	0.4	0.48	0.53	0.56	0.63	
Polkhan	8.1	10.5	11.7	12.5	13.7	14.4	
Bonkooh	2.9	3.5	4	4.5	5.1	5.5	
Dashtbal	0.54	0.75	0.87	0.95	1	1.1	
Dodehak	3.4	5.2	5.62	5.69	8.78	10.84	
Yalfan	0.002	0.003	0.004	0.004	0.005	0.005	
Ghamsar	0.011	0.012	0.012	0.012	0.013	0.013	

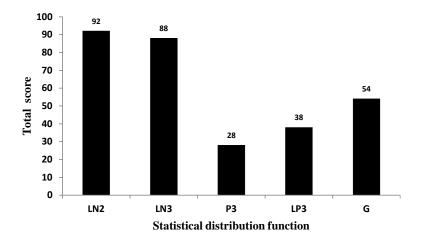


Fig. 3. Sum of scores calculated from annual low-flow series for different statistical distribution in L-moment

# 4. Conclusion

Low-flow regionalization is a critical component of water-resource planning and management in ungauged or poorly gauged arid and semi-arid catchment areas. The application of physically based rainfall-runoff approach for a large basin may not be practicable due to the large spatial variation both in basin characteristics and in the hydrological inputs. Our study indicates that the statistical approach presents a practical solution for low-flow regionalization with reasonably accurate outcomes in the Iranian Central Plateau River Basin, Iran.

We performed this study to select a set of adequate probability observed distributions for modeling low-flow series in the Iranian Central Plateau River Basin. Our results showed that the L-moments method is extremely suitable to assess goodness-of fit as well as regional analysis. Compared to the conventional method, Lmoments has several advantages. Analysis of 20 stations in the study area indicated that the Pearson type III, log Pearson type III, and Gumbel distributions estimations for annual low-flow series were better than the other distributions. Thus, these probability distributions can be recommended for setting low flows at these stations. However, these distributions are not necessarily suitable for places in other arid and semi-arid regions of Iran or other geographically similar country. Examinations and assessments similar to those performed in the present research area would be needed to identify the best distributions and to select the minimum flow frequency analysis method in other regions. These methods can also be applied to provide a scientific basis for the delineation of low flows for the effective management of competing water uses and conservation of instream values.

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## References

- Bayazit, M., B. Önoz, 2002. LL-moments for estimating low flow quantiles / Estimation des quantiles d'étiage grâce aux LL-moments. Hydrological Sciences Journal, 47: 707-720.
- Caruso, B.S., 2000. Evaluation of low-flow frequency analysis methods. Journal of Hydrology (NZ), 39; 19-47.
- Chen, Y.D., G. Huang, Q. Shao, C. Xu, 2006. Regional analysis of low flow using L-moments for Dongjiang basin, South China. Hydrological Sciences Journal, 51; 1051-1064.
- Chowdhury, J.U., J.R. Stedinger, L. Lu, 1991. Goodnessof-fit tests for regional GEV flood distributions. Water Resource Research, 27; 1756-1776.
- Daviau, J.L., K. Adamowski, G.G. Patry, 2000. Regional flood frequency analysis using GIS, L-moment and

- geostatistical methods. Hydrological Processes, 14; 2731-2753.
- Delleur, J.W., A.R. Rao, J.M. Bell, 1988. Criteria for the determination of minimum streamflows. Technical report CE-HSE-88-6, School of Civil Enginieering, Purdue University, West Lafayyete, Ind.
- Dodangeh, E., S. Soltani, A. Sarhadi, J.T. Shiau, 2013. Application of L-moments and Bayesian inference for low-flow regionalization in Sefidroud basin, Iran. Hydrological Processes, 35; 1-14.
- Du, T., L. Xiong, Ch.Y. Xu, Ch.J. Gippel, S. Guo, P. Liu, 2015. Return period and risk analysis of nonstationary low-flow series under climate change. Journal of Hydrology, 527; 234–250.
- Eslamian, S.S., S. Chavoshi, 2003. The Application of L-moments Theory in Flood Frequency Analysis of Central Catchments of Iran. JWSS, 7; 1-17.
- Eslamian, S.S., H. Feizi, 2007. Maximum monthly rainfall analysis using L-moments for an arid region in Isfahan province, Iran. Journal of Applied Meteorological and Climatology, 46; 494-503.
- Greenwood, J.A., J.M. Landwehr, N.C. Matalas, J.R. Wallis, 1979. Probability Weighted Moments: Definitions and Relation to Parameters of Several Distributions Expressible in Inverse Form. Water Resource Research, 15; 1049-1054.
- Gustard, A., Young, A.E., Rees, G., Holmes, M., 2004. Low flows, drought and operational hydrology. In: Tallaksen, L.M., van Lanen, H.A.J. (Eds.), Hydrological Drought: Processes and Estimation Methods for Streamflow and Groundwater Developments in Water Science, vol. 48. Elsevier, NL.
- Hosking, J.R.M., 1990. L-moments: analysis and estimation of distributions using linear combinations of order statistics. Journal of Royal Statistical Society, 52; 105-124.
- Hosking, J.R.M., J.R. Wallis, 1993. Some statistics useful in regional frequency analysis. Water Resource Research, 29; 271-281.
- Hosking, J.R.M., J.R. Wallis, 1997. Regional frequency analysis: an approach based on L-moments. Cambridge University Press, Cambridge, UK.
- Iran Water Resources Management Company, 2013. Low flow data of Iranian Central Plateau River Basin.
- Jingyi, Z., M.J. Hall, 2004. Regional Flood Frequency Analysis for the Gan-Ming River Basin in China. Journal of Hydrology, 296; 98-117.
- Keshtkar, A.R., A. Salajegheh, M. Najafi Hajivar, 2012. Flood flow frequency model selection using L-moment method in arid and semi-arid regions of Iran. Desert, 17; 41-48.
- Khandi, S., H. Kachroo, R.K. Gunasekara, 2000. Comparison of Annual Maximum Series and Partial Duration of Homogeneous Regions. Hydrological Science Journal, 45: 437-447.
- Kjeldsen, T.R., J.C. Smithers, R.E. Schulze, 2001. Regional Flood Frequency Analysis in the KwaZulu-Natal Province, South Africa, Using the Index-Flood Method. Journal of Hydrology, 255; 194-211.
- Kroll, C.N., Vogel, R.M., 2002. Probability distribution of low streamflow series. Journal of Hydrological Engineering, ASCE, 7; 137–146.

- Kumar, R., Chatterjee, C., Kumar, S., Lohani, A.K., Singh, R.D., 2003. Development of regional flood frequency relationships using L-moments for Middle Ganga Plains Subzone 1(f) of India. Water Resource Management, 17: 243–257.
- Laaha G., G. Bloschi, 2006. A comparison of low flow regionalisation methods catchment grouping. Journal of Hydrology, 323; 193–214.
- Lim, Y.H., Lye, L.M., 2003. Regional flood estimation for ungauged basins in Sarawak, Malaysia. Hydrological Science Journal, 48; 79–94.
- Malekinezhad, H., H.P. Nachtnebel, A. Klik, 2011. Regionalization Approach for Extreme Flood Analysis Using L-moments. Journal of Agricultural Science and Technology, 13; 1183-1196.
- Matalas, N.C., 1963. Probability distribution of low flows. Report 434-A, U.S. Geological Survey, Washington, D.C.
- Minocha, V.K., 2003. Discussion of Probability distribution of low streamflow series by C.N. Kroll & R.M. Vogel. Journal of Hydrologic Engineering, ASCE, 8; 297.
- McCuen, R.H., 2003. Modeling hydrologic change: Statistical methods. CRC Press, Florida, 433p.
- Modarres, R., 2008. Regional Frequency Distribution Type of Low Flow in North of Iran by L-moments. Water Resources Management, 22; 823-841.
- Modarres, R. 2009. Regional Dry Spells Frequency Analysis by L-Moment and Multivariate Analysis. Water Resources Management, 24; 2365-2380.
- Pandey, M.D., Gelder, P.H., Vrijling, J.K., 2001. Assessment of an L-kurtosis-based criterion for quantile estimation. Journal of Hydrologic Engineering ASCE, 6: 284–291.
- Parida, B.P., Kachroo, R.K., Shrestha, D.B., 1998.
   Regional flood frequency analysis of Mahi-Sabarmati
   Basin (Subzone 3-a) using index flood procedure with
   L-moments. Water Resources Management, 12; 1–12.
- Pearson, C.P., 1995. Regional frequency analysis of low flow in New Zealand rivers. Journal of Hydrology (NZ), 33: 94-122.
- Peel, M.C., T.A. McMahon, B.L. Finlayson, 2004. Continental Differences in the Variability of Annual Runoff-Update and Reassessment. Journal of Hydrology, 295; 185-197.
- Peng, S., C. Xi, Q. Si-min, Y. Tao, Z. Zhi-cai, M. Jianliang, 2009. Regional Frequency Analysis of Low Flow Based on L-Moments in Karst Area, Southwest China. Joint International Convention of 8th IAHS Scientific Assembly and 37th IAH Congress, Water: A vital resource under stress How Science can help, September 6-12, Hyderabad, India.
- Rostami, R., 2013. Regional Flood Frequency Analysis Based on L-Moment Approach (Case Study West

- Azarbayjan Basins). Journal of Civil Engineering and Urbanism, 3; 107-113.
- Salajegheh, A., A.R. Keshtkar, S. Dalfardi, 2008. To study of the appropriate probability distributions for annual maximum series using L-moment method in arid and semi-arid regions of Iran. XIII World Water Congress, 1-4 september, Montpellier, France.
- Sarhadi, A., S. Soltani, R. Modares, 2008. Regional frequency analysis for low flow using L-moment method in Jiroft region. Iranian Water Research Journal (In Persian), 3; 45-54.
- Sawaske, S.R., D.R. Freyberg, 2014. An analysis of trends in baseflow recession and low-flows in rain-dominated coastal streams of the pacific coast. Journal of Hydrology, 519; 599–610.
- Shi, P., X. Chen, S. Qu, Zh. Zhang, J. Ma, 2010. Regional Frequency Analysis of Low Flow Based on L Moments: Case Study in Karst Area, Southwest China. Journal of Hydrologic Engineering, ASCE, 15:370-377.
- Smakhtin, V.U., 2001. Low flow hydrology: a review. Journal of Hydrology, 240; 147–186.
- Smithers, J.C., R.E. Schulze, 2000. A Methodology for the Estimation of Short Duration Design Storms in South's Africa Using a Regional Approach Based on Lmoments. Journal of Hydrology, 241; 42-52.
- Staudinger M, J. Seibert, 2014. Predictability of low flow An assessment with simulation experiments. Journal of Hydrology, 519: 1383–1393.
- Stedinger, J.R., R.M. Vogel, F. Foufula-Georgiou, 1993. Frequency Analysis of Extreme Events, In Handbook of Applied Hydrology, Ch18, Editor-in-chief, D.A. Maidment, Mc Grow-Hill, Newyork.PP.1-66.
- Thomas, B., G. Lishchied, J. Stiedl, O. Dietrich, 2015. Long term shift of low flows predictors in small lowland catchments of Northeast Germany. Journal of Hydrology, 521; 508–519.
- Vogel, R.M., C.N. Kroll, 1989. Low flow frequency analysis using probability plot correlation coefficients. Journal of Water Resources Planning and Management, ASCE, 115; 338-357.
- Vogel, R.M., N.M. Fennessey, 1993. L-moments diagrams should replace product moment diagrams. Water Resource Research, 29; 1745-1752.
- Vogel, R.M., W.O. Thomas, T.A. McMahon, 1993. Flood flow frequency model selection in Southwestern U.S.A. Journal of Water Resources Planning and management, ASCE, 119; 353-366.
- Vogel, R., I. Wilson, 1996. Probability Distribution of Annual Maximum, Mean & Minimum Stream Flows in the United States. Journal of Hydrologic Engineering, 12; 69-76.
- Yurekli, K., A. Kurunc, S. Gul, 2005. Frequency Analysis of Low Flow Series from Çekerek Stream Basin. Tarim Bilimleri Dergisi, 11; 72-77.