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Flood Flow Frequency Model Selection Using L-moment Method in Arid and Semi Arid Regions of Iran

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Abstract

Statistical frequency analysis is the most common procedure for the analysis of flood data at a gauged location that in first step it is needed to select a model to represent the population. Among them, the central moment has been the most common and widely used, and with the using of computers, the application of the maximum likelihood has increased. This research was carried out in order to recognition of suitable probability distributions with pervious common methods. In order to investigate of suitable probability distribution for flood flow, using L-moment method through the existing hydrometric stations in the region, 17 hydrometric stations were selected for peak discharges data studies. According to results of this research for peak discharge, LP3 distribution and ordinary moment method, P3 distribution and L-moment method, LN2 and LN3 distributions and ordinary method have been suitable distinguished for 53%, 35%, 6% and 6% of stations, respectively. We concluded that L-moment method is suitable to determine peak series probability distributions in the Iran central plateau and P3 is the best probability distribution for modeling peak series in this region.

Keywords: Flood flow; Frequency distribution function; Linear moment; Maximum likelihood; Ordinary moment

1. Introduction

Hydrological frequency analysis is one of the essential tasks in hydrological engineering design. It is the work of determining the magnitude of hydrological variables corresponding to given frequencies or recurrence intervals (McCuen, 2003). Procedures involved in frequency analysis include: collection a random sample of the interested hydrological variables; finding the best-fit-distribution for the samples by a good-of-fit (GOF) test or other appropriate methods; and determining the magnitude of the hydrological variable corresponding to a given probability of exceedance using the best-fit-distribution. Chow (1951) proposed the following general equation for hydrologic frequency analysis of a random variable X with mean μ and standard deviation σ:

$$X_T = \mu + K_T\sigma \qquad (1)$$

Where K_T, the frequency factor is a function of the return period T and is dependent on the distribution type of X. Frequency factors of distributions commonly used in hydrologic frequency analysis are themselves variables and their distributions have been identified (Kite, 1988). Upon collection of a random sample, one must decide the type of distribution which best characterizes the random sample and determine the corresponding K_T value.

Two GOF tests, namely the chi-square test and the Kolmogorov-Smirnov test, are often used for the selection of probability distributions for hydrological variables (Haan, 2002). Another method of goodness-of-fit test is the method based on ordinary moment ratio diagrams (D' Agostino and Stephen, 1986). Moment ratios are unique properties of probability distributions and sample moment ratios of ordinary skewness and kurtosis have

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been used for selection of probability distribution (Kottegoda, 1980 and D'Agostino and Stephen, 1986). In recent years there have been many applications of L-moments diagrams for selecting various distributional alternatives in a region and the skewness and kurtosis L-moment-ratio diagram (LMRD) was suggested as a useful tool for discrimination between candidate distributions (Hosking and Wallis, 1987; Hosking, 1990; Hosking and Wallis, 1993; Vogel and Fennesset, 1993; Hosking and Wallis, 1997 and Peel et al., 2001). The L-moments uniquely define the distribution if the mean of distribution exists, and the L-skewness and L-kurtosis are much less biased than the ordinary skewness and kurtosis (Hosking and Wallis, 1997).

Estimation of flood flow is often required for watersheds with insufficient or nonexistent hydrometric information particularly in arid and semi-arid regions. Because parametric methods require a number of assumptions, nonparametric methods have been investigated as alternative methods. L-moment diagrams and associated goodness-of-fit procedure (Wallis, 1988; Hosking, 1990; Chowdhury et al., 1991; Pearson, 1992; Vogel et al., 1993a; Daviau et al., 2000; Peel et al., 2004; Yurekli et al., 2005; Chen et al., 2006; Eslamian and Feizi, 2007 and Salajegheh et al., 2008) have been advocated for evaluating the suitability of selecting various distributional alternatives for modeling flows in a region. For example Wallis (1988) found an L-moment diagram useful for rejecting Jain and Singh's (1987) conclusion that annual maximum flood flows at 44 sites were well approximated by a Gumbel distribution and for suggesting a general extreme value (GEV) distribution instead. Vogel et al., (1993b) used L-moment diagrams to show that two and three-parameter log-normal models (LN2 and LN3), the LP3 and the GEV distributions were all acceptable models of flood flows in the southwestern United States. Gholami et al., (2001) used L-moment diagrams to show that the Gumbel distribution was acceptable for annual maximum series (AMS) in the north of Iran.

Base on the climate conditions and annual average precipitation, Iran is one of the arid and semi-arid countries of the world, which is mostly encountered with lack of water. Most of the rivers in arid regions of Iran are seasonal and their flood flows may become unavailable during a short time of rainfall seasons and because of some special geological problems of this region most of the permanent rivers contain saline water and are useless. The objective of the present study is to introduce and evaluate the suitable probability distributions for modeling peak flows in arid and semi-arid regions of Iran.

2. Materials and Methods

2.1. Study area

The Iran central plateau watershed where is located in center of Iran, has considered as a arid and semi-arid region of Iran. The Iran central plateau watershed is 854000 Km^2 in area and has seven sub watersheds and several catchments in different size. For the purpose of this research 17 catchments were chosen where permanent river and hydrometric station had. The selection was based on Ministry of Agriculture information that no significant abstraction exist in the contributing catchments upstream of these stations. The location of the hydrometric stations utilized in this research is given in table 1 and the spatial distribution shown in fig.1.

Table 1. Main characteristics of stations under investigation and data series analyzed

Station	River	Longitude (E)	Latitude (N)	Altitude (m)	Catchment area (Km^2)
Sarab hendeh	Golpaiegan	50	33 21	2000	817
Abgarm	Kharood	48 18	33 45	1560	2450
Dehsomee	Kordan	50 50	35 57	1410	360
Sira	Karaj	51 09	36 02	1790	725
Dodahak	Ghomrood	50 34	34 28	1470	8851
Chamriz	Kor	52 07	30 28	1800	3390
Baft	Soltani	56 32	29 05	2000	935
Safarzadeh	Halilrood	57 33	28 52	920	8420
Dehrood	Shoor	57 44	28 52	1000	1321
Adoori	Talango	58 07	28 58	1690	276
Jirofto	Haftkoosk	57 11	29 34	2600	225
Ghariatolarab	Chary	56 02	29 37	1853	173
Godarzarch	Abbakhsha	56 34	29 34	2200	1144
Namrood	Namrood	52 39	35 43	1810	587
Simindasht	Delichai	52 31	35 31	1435	2254
Bonkooh	Hablerood	52 52	35 18	1000	3209
Senobar	Shastdare	59 06	35 26	1760	152

Fig. 1. Location of the study area

2.2. Methods

In this research, several methods were carried out in order to propose the most suitable probability distributions for peak series. From the hydrometric stations existing in the watershed, 17 stations were selected for analysis. The peak flows data were available for 17 gauging stations, with stream flow records of 20 years or more. Most of the data were recorded between 1972 and 2000.

Then, data were analyzed to choose an identical data period. Therefore, the peak series data from 1971 to 1998 were taken as the 25-year data period of each station.

The missing data were generated and completed using regression relationships between the stations. For each distribution, the values of residual sum of squares (RSS) were calculated using the ordinary moment and L-moment methods. The RSS values of the two methods were compared, suitable distributions for each station were chosen according to the lowest RSS. The best of probability distribution was applied to estimate *T*-year peak series.

2.3. The ordinary moment ratio diagram

Parameters of a probability distribution can be expressed in terms of its moments (μ'_r) or central moments (μ_r) defined as

$$\mu'_r = E\,(X^r) \qquad (2a)$$

$$\mu_r = E\,(X - \mu^1{}_r)^r \qquad (2b)$$

For r= 1, 2 therefore, the shape of a probability distribution can be characterized by the moments of that distribution. This is achieved using the relationships between the standardized coefficients of skewness ($\sqrt{\beta_1}$) and kurtosis (β_2) which are defined as

$$\sqrt{\beta_1} = \mu_3/(\mu_2)^{3/2} \qquad (3a)$$

$$\beta_2 = \mu_4/(\mu_2)^2 \qquad (3b)$$

Theoretical $\sqrt{\beta_1}\sim$ β_2 relationships of various distributions (fig. 2) are known as the moment-ratio diagram. Hereafter, the standardized

moments ($\sqrt{\beta_1}$) and (β_2) will be referred to as the skewness and kurtosis, respectively.
Given a random sample $\{x_1, x_2, \ldots, x_n\}$, the sample estimates of $\sqrt{\beta_1}$ and β_2 are defiened as

$$\sqrt{b_1} = \frac{m_3}{m^3} \tag{4a}$$

$$b_2 = \frac{m_4}{m^4} \tag{4b}$$

Where

$$m_r = \frac{1}{n}\sum_{j=1}^{n}([x_1] - \bar{x})^2 \tag{4c}$$

$$S = \sqrt{m_2} \tag{4d}$$

Sample estimates of ($\sqrt{(\beta_1 1)}$) and (β_2) do not always coincide with the theoretical points or curves of their parent distributions, and method of GOF test using ($\sqrt{b_1}$ b_2) have thus been developed for normal distribution (D'Agostino and Pearson, 1973 and Bowman and Shenton, 1975). Bowman and Shenton (1986) developed acceptance regions for a($\sqrt{(\beta_1 1)}$, β_2) based GOF test using stochastic simulation. The sample moments ($\sqrt{b_1}$ b_2) tend to have high variances, especially when the sample size is small. Therefore, it is often difficult to distinguish among candidate distributions. Even though the joint distribution of the ordinary sample skewness and sample kurtosis is sympototically normal, such asympototic property is a poor approximation in small and moderately samples, particularly when the underlying distribution is even moderately skew (Hosking and Wallis, 1997). Using stochastic simulation, Wu (2005) studied the distribution of ($\sqrt{b_1}$ b_2) for normal distribution with respect to sample sizes varying from 20 to 1000. As can be seen in fig. 2, for smaller sample sizes (for example n=20) the distribution of $\sqrt{b_1}$ given b_2 changes from unimodality to bimodality as b_2 increases. Thus, closed curves encompassing certain percentages of ($\sqrt{b_1}$ b_2) samples of the normal distribution are difficult to be expressed by mathematical equations when the sample size is small.

L-moment analysis

L-moment is alternative system of describing the shapes of probability distributions. L-moments are linear combinations of order statistics which are robust to outliers and virtually unbiased for small samples, making them suitable for flood frequency analysis, including identification of distribution and parameter estimation (Hosking, 1990 and Hosking and Wallis, 1993). L-moments are defined as linear combinations of probability weighted moments (PWM):

$$\beta_r = E\{X[F_x(x)^r]\} \tag{5}$$

Where β_r is the rth order PWM and $F_x(x)$ is the cdf of X. When r=0, β_0 is the mean stream flow. Hence a sample estimate of the first PWM, which we term b_0 is simply the sample mean. Nevertheless, unbiased estimators are often preferred in goodness-of-fit evaluations such as L-moment diagrams. Unbiased sample estimates of the PWMs, for any distribution can be computed from

$$b = \frac{1}{n}\sum_{j=1}^{n} x_j \tag{6a}$$

$$b_1 = \sum_{j=1}^{n-1}\left[\frac{(n-j)}{n(n-1)}\right]x_j \tag{6b}$$

$$b_2 = \sum_{j=1}^{n-2}\left[\frac{(n-j)(n-j-1)}{n(n-1)(n-2)}\right]x_j \tag{6c}$$

$$b_3 = \sum_{j=1}^{n-3}\left[\frac{(n-j)(n-j-1)(n-j-2)}{n(n-1)(n-2)(n-3)}\right]x_j \tag{6d}$$

Where x_j represents the ordered stream flows with x_1 being the largest observation and x_n the smallest. The PWM estimators in eqn. (6) can be more generally described using

$$b_r = \frac{1}{n}\sum_{j=1}^{n-r}\left[\frac{\binom{n-j}{r}}{\binom{n-1}{r}}\right]x_j \tag{7}$$

For a random variable X with quantile function $x(u)$, Hosking and Wallis (1997) defined the L-moments (λ_r, r = 1,2, …) as

$$\lambda_r = \int_0^1 x(u)p^*_{r-1}(u)d_u \tag{8a}$$

Where

$$p^*_r = \sum_{k=0}^{r} p^*_{r,k}u_k, \quad r=0,1,2,\ldots \tag{8b}$$

$$p^*_{r,k} = (-1)^{r-k}\binom{r}{k}\binom{r+k}{k} = \frac{(-1)^{r-k}(r+k)!}{(k!)^2(r-k)!} \tag{8c}$$

The L-moments can also be expressed in terms of the probability weighted moments defined by

Greenwood et al., (1979), and for any distribution, the first four L-moments are easily computed from PWMs using

$\lambda_1 = \beta_0$ (9a)

$\lambda_2 = 2\beta_1 - \beta_0$ (9b)

$\lambda_3 = 6\beta_2 - 6\beta_1 - \beta_0$ (9c)

$\lambda_4 = 30\beta_2 - 12\beta_1 - \beta_0$ (9d)

Where β_r, r = 0,1,2,…, are probability weighted moments defined by

$$\beta_r = \int_0^1 x(u)u^r du \quad (10)$$

In terms of linear combination of order statistics, the L-moments can also be expressed by

$\lambda_1 =$ (11a)

$$\lambda_2 = \frac{1}{2} E(X_{2:2} - X_{1:2}) \quad (11b)$$

$$\lambda_3 = \frac{1}{3} E(X_{3:3} - X_{2:3} - X_{1:3}) \quad (11c)$$

$$\lambda_4 = \frac{1}{4} E(X_{4:4} - X_{3:4} - X_{2:4} - X_{1:4}) \quad (11d)$$

Where $X_{k:n}$ is the K-th order statistic from a random sample of size n. similar to the ordinary moment ratios, the L-moment ratios are defined by

$$\tau_r = \frac{\lambda_r}{\lambda_2} \quad (12)$$

Theoretical relationships between L-skewness (τ_3) and L-kurtosis (τ_4), i.e. the L-moment ratio diagram of several probability distributions have been given by Hosking (1990 and 1991) and can be used to distinguish different probability distributions.

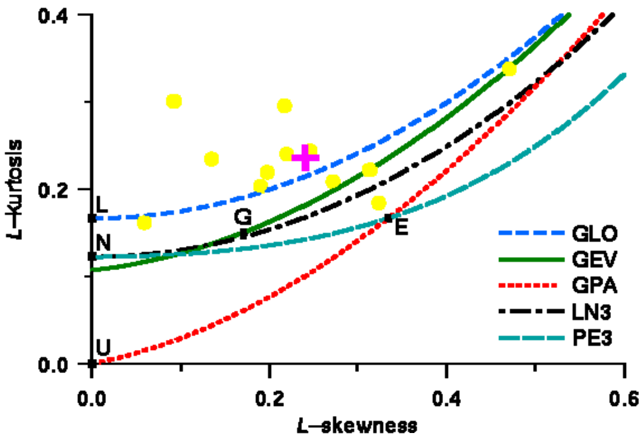

Fig. 2. L-moment ratio diagram of various distributions

L-moment ratio diagrams

Analogous to the product moment ratio; coefficient of variation $C_v = \sigma / \mu$, skewness γ and kurtosis $_k$, Hosking (1990) define the L-moment ratios

(13a)

$$\tau_3 = \frac{\lambda_3}{\lambda_2} = L - skewness \quad (13b)$$

$$\tau_4 = \frac{\lambda_4}{\lambda_2} = L - kurtosis \quad (13c)$$

Where λ, r = 1, …, 4 are the first four L-moments and τ_2, τ_3 and τ_4 are the L-coefficient of variation (L-C_v), L-skewness and L-kurtosis, respectively. The first L-moment λ_1 is equal to the mean stream flow μ; hence it is a measure of location. Hosking (1990) shows that λ_2; τ_3 and τ_4 can be thought of as measures of a distribution's scale, skewness and kurtosis, respectively, analogous to the moment's σ, γ and $_k$ respectively.

3. Results

According to results of this research for peak discharge and based on computed residual sum of squares (RSS), LP3 distributions and ordinary moment method, P3 distribution and L-moment method, LN2 and LN3 distributions and ordinary moment method, have been suitable distinguished for 53%, 35%, 6% and 6% of stations, respectively (Table 2). Estimated *T*-year peak series using the best of probability distribution have shown in table 3.

Table 2. Suitable probability distributions and methods for peak series

Suitable Method	Suitable Probability Distribution	RSS	Station
L-moment	P3	40	Sarab hendeh
Ordinary moment	LP3	15	Abgarm
Ordinary moment	LN3	5.9	Dehsomee
Ordinary moment	LP3	8.2	Sira
Ordinary moment	LP3	18	Dodahak
Ordinary moment	LP3	8.9	Chamriz
Ordinary moment	LP3	16.9	Baft
Ordinary moment	LP3	10.5	Safarzadeh
Ordinary moment	LP3	9.4	Dehrood
L-moment	P3	34.4	Adoori
L-moment	P3	11.2	Jirofto
L-moment	P3	10.6	Ghariatolarab
Ordinary moment	LP3	14.1	Godarzarch
L-moment	P3	24.6	Namrood
L-moment	P3	22.8	Simindasht
Ordinary moment	LP3	12.1	Bonkooh
Ordinary moment	LN2	15.3	Senobar

Table 3. Estimated *T*-Year peak series

T-Years						Station
100	50	20	10	5	2	
205	188	162	144	120	80	Sarab hendeh
130	121	107	99	90	73	Abgarm
128	116	101	89	77	57	Dehsomee
214	192	159	142	120	87	Sira
147	132	111	99	84	62	Dodahak
527	500	457	433	398	341	Chamriz
238	171	144	108	82	47	Baft
821	719	571	496	403	273	Safarzadeh
495	460	408	379	339	275	Dehrood
181	171	157	143	127	100	Adoori
48	44	41	35	31	22	Jirofto
20	20	19	18	17	16	Ghariatolarab
105	95	80	71	59	41	Godarzarch
84	77	68	60	51	30	Namrood
78	74	66	60	52	39	Simindasht
71	67	57	48	44	33	Bonkooh
266	237	198	169	140	97	Senobar

For two methods sum of scores obtained in order to better comparison between distributions. First rank was gave to each distribution that RSS estimated was lowest and fifth rank for each distribution that RSS estimated was highest and in equal scores, ranking was similar. Finally sum of scores were computed for any distribution that according this method distribution was best that was accepted lowest score (Fig. 3).

4. Conclusion

The goal of this study was to select a set of suitable probability observed distributions for modeling peak series in arid and semi-arid regions of Iran. L-moment diagrams revealed that the Pearson type 3 (P3), log Pearson type 3 (LP3), Gumbel (G) distributions all provide acceptable approximations to the distribution of peak series in the study area (Figure 1), which means that other three and two-parameter alternatives LN3 and LN2 are not acceptable for the most parts of the study region. These results are similar to the other studies such as Pearson et al., 1991; Vogel et al., 1993b; Vogel and Wilson, 1996; Adamowski, 2000 and Kjeldson et al; 2002 that were used L-moment diagrams for choosing various distributional alternatives for annual maximum data in a region. Of all the models evaluated, the P3 distributions probably provide the best description of the distribution of peak series across the entire Iran central plateau, however, separation of central plateau in to broad homogeneous regions can improve our ability to discriminate among potential flood flow frequency models such that the P3 distribution provides the best approximation to the distribution of peak series in the most of this region. Of the models tested, the LP3, P3 and LN2 distributions provides the best approximation to the distribution of peak series using ordinary moment method. Finally we concluded that L-moment method is suitable to determine peak series probability distributions in the Iran central plateau and P3 is the best probability distribution for modeling peak series in arid and semi-arid regions of Iran.

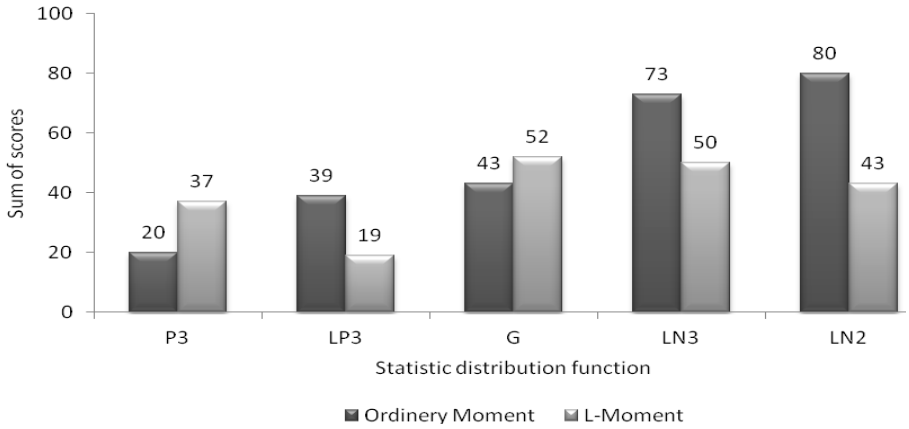

Fig. 3. Sum of scores obtained from peak series for different statistical distribution in L-moment and ordinary moment

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